Atomic and Nuclear Physics

Atomic Shell

Fabry-Pérot Interferometer, Determination of the Bohr Magneton

SPECTROSCOPY WITH A FABRY-PÉROT ETALON

- **Experimental introduction to the Fabry-Pérot interferometer using the example of the normal Zeeman effect**
- **Measuring the interference rings of the Fabry-Pérot etalon as a function of the external magnetic field**
- **Determination of the Bohr Magneton**

UE5020900

09/24 UD

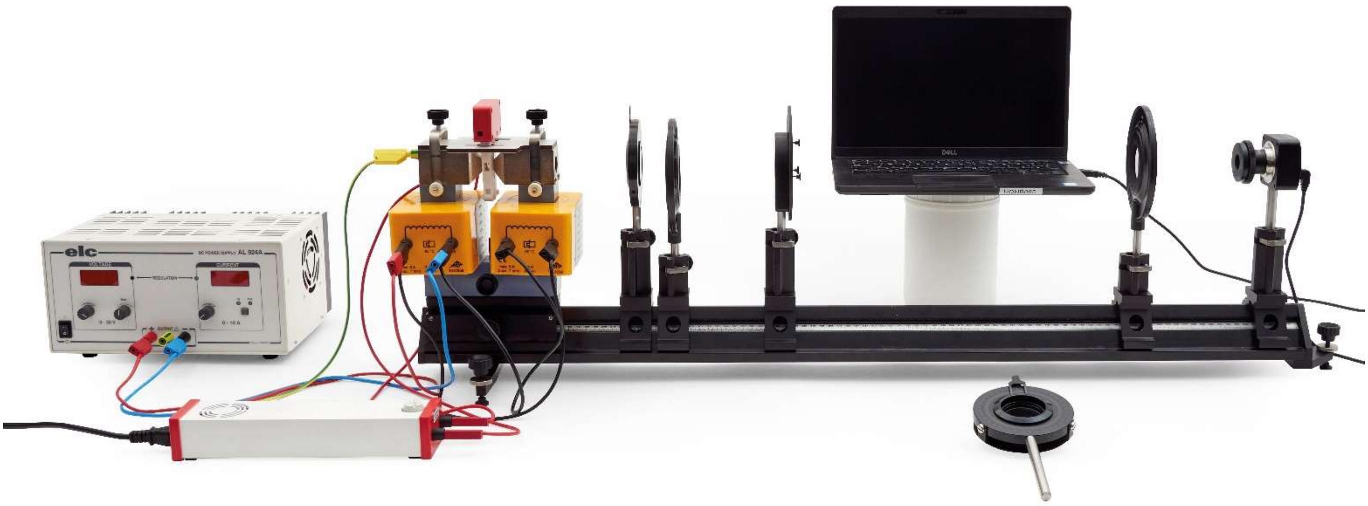

Fig. 1: Experimental setup for the normal Zeeman effect in longitudinal configuration

GENERAL PRINCIPLES

The Fabry-Pérot interferometer, developed by its namesakes Charles Fabry and Alfred Pérot, is an optical resonator consisting of two semi-transparent mirrors. A Fabry-Pérot interferometer with a fixed distance between the mirrors is known as a Fabry-Pérot etalon. As it is designed to fulfill the resonance condition for a specific wavelength, the etalon also acts as an optical filter. An incident light beam is reflected several times in the etalon so that the light beams transmitted with each reflection interfere with each other. This multi-beam interference produces an intensity distribution in transmission with narrow maxima and broad minima. Together with the high interference order at correspondingly large resonator dimensions, this results in a high optical quality and correspondingly high resolution. This means that small spectral splittings, such as those present in the normal Zeeman effect at the red Cd line (λ = 643.8 nm, $\Delta\lambda$ = 0.0068 nm at B = 350 mT), can be resolved.

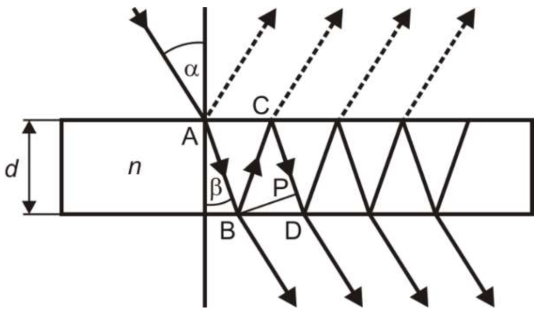

Fig. 2: Beam path in the Fabry-Pérot etalon

A theoretical description of the normal Zeeman effect can be found in the instructions for experiment UE5020850, in which the doublet and triplet splitting is investigated qualitatively.

The focus of this experiment is on spectroscopy with a Fabry-Pérot etalon. The Fabry-Pérot etalon is positioned in front of the camera together with imaging optics, which is used to observe Zeeman splitting. When the light from the cadmium lamp passes through the Fabry-Pérot etalon, interference rings are created which, like the spectral line, split depending on the external magnetic field and are imaged onto the camera by the optics. Observation parallel or perpendicular to the external magnetic field is made possible by a rotating electromagnet.

The Fabry-Pérot etalon consists of a quartz glass plate with a semi-reflective mirror coating of high reflectivity on both sides (Fig. 2). In this case, the etalon is designed in such a way that the resonance condition for the wavelength λ = 643,8 nm of the red Cd line is fulfilled. In this sense, the etalon also acts as an optical filter. The thickness *d*, the refractive index *n* and the reflection coefficient *R* of the etalon are as follows:

$$(1)\quad \begin{aligned} d &= 4\,\text{mm} \\ n &= 1.4567 \\ R &= 0.85 \end{aligned}$$

An incident light beam is reflected several times in the etalon. The light beams transmitted during each reflection interfere with each other. The path difference Δs between two neighboring transmitted light beams, e.g. the light beams emerging at points B and D in Fig. 2, is:

$$(2)\quad \Delta s = n \cdot \left(\overline{\text{BC}} + \overline{\text{CP}}\right).$$

From

$$(3)\quad \overline{\text{CP}} = \overline{\text{BC}} \cdot \cos(2 \cdot \beta),$$

$$(4)\quad d = \overline{\text{BC}} \cdot \cos(\beta),$$

Snellius' law of refraction ($n_{\text{air}} \approx 1$)

$$(5)\quad \sin(\alpha) = n \cdot \sin(\beta)$$

and the addition theorems

$$(6)\quad \begin{aligned} \cos(\beta) &= \sqrt{1 - \sin^2(\beta)} \\ \cos(2 \cdot \beta) &= 1 - 2 \cdot \sin^2(\beta) \end{aligned}$$

the path difference results in

$$(7)\quad \Delta s = 2 \cdot d \cdot \sqrt{n^2 - \sin^2(\alpha)} = 2 \cdot d \cdot n \cdot \cos(\beta)$$

and from this, the condition for the existence of interference maxima:

$$(8)\quad k \cdot \lambda = 2 \cdot d \cdot \sqrt{n^2 - \sin^2(\alpha_k)} = 2 \cdot d \cdot n \cdot \cos(\beta_k).$$

k: Whole number, interference order
α_k: Incidence angle of the *k*th interference order
β_k: Refraction angle of the *k*th interference order

Overall, an interference pattern of concentric rings is generated. The refraction at the boundary surfaces of the glass plate of the Fabry-Pérot etalon can be neglected as it only shifts the interference pattern in parallel. Therefore, the refraction angle β is replaced by the incidence angle α, and the interference condition (8) results in

$$(9)\quad k \cdot \lambda = 2 \cdot d \cdot n \cdot \cos(\alpha_k) \approx 2 \cdot d \cdot n \cdot \left(1 - \frac{\alpha_k^{\,2}}{2}\right),$$

with the expansion $\cos(x) \approx (1 - x^2 / 2)$ of the cosine function.

The interference pattern is imaged onto the camera using the convex lens (Fig. 3). The following relationship exists between the angle α_k at which the *k*th order interference ring appears, the radius r_k of the *k*th order interference ring and the focal length *f* of the lens (Fig. 3):

$$(10)\quad r_k = f \cdot \tan(\alpha_k) \approx f \cdot \alpha_k,$$

with the small angle approximation $\tan(x) \approx x$. From equation (9) follows for the interference order *k* and the angle α_k

$$(11)\quad k = k_0 \cdot \cos(\alpha_k) \approx k_0 \cdot \left(1 - \frac{\alpha_k^{\,2}}{2}\right) \text{ with } k_0 = \frac{2 \cdot d \cdot n}{\lambda}$$

and

$$(12)\quad \alpha_k = \sqrt{\frac{2 \cdot (k_0 - k)}{k_0}}.$$

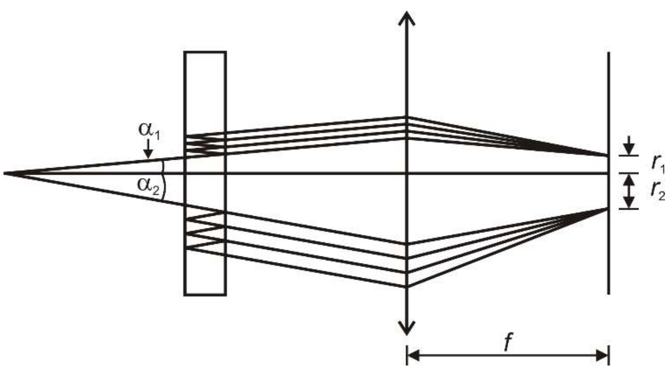

Fig. 3: Imaging the interference rings of the Fabry-Pérot etalon onto the digital camera

According to equation (11), because of $|\cos(\alpha_k)| \leq 1$, the interference order k is maximum for $\alpha_k = 0$, i.e. in the center of the interference rings, and corresponds to the parameter k_0, which is generally not a whole number. Since the interference rings are counted from the center in the experiment, the interference order k is indexed with a whole number j, which identifies the kth interference order with the jth interference ring counted from the center, in generalization of the parameter k_0 already introduced.

The first bright interference ring with order k_1 appears according to equation (12) at the angle

$$(13)\quad \alpha_{k_1} = \sqrt{\frac{2\cdot(k_0 - k_1)}{k_0}},$$

where k_1 is the next whole number that is smaller than k_0. As k_0 is generally not a whole number, the difference $k_0 - k_1$ is less than 1. Therefore, a parameter ε is defined as follows:

$$(14)\quad \varepsilon := k_0 - k_1 \text{ mit } 0 < \varepsilon < 1$$

For all interference rings with $j \geq 2$, the order number k_j is decreased by 1 in each case, so that for the interference order of the jth interference ring counted from the center the following is generally true:

$$(15)\quad k_j = (k_0 - \varepsilon) - (j - 1)$$

For $j = 1$, equation (15) just corresponds to the definition of ε from equation (14). Substituting equations (12) (with $k = k_j$) and (15) into equation (10) results in

$$(16)\quad r_j = \sqrt{\frac{2\cdot f^2}{k_0}}\cdot\sqrt{(j-1)+\varepsilon},$$

where, for the sake of simplicity, $r_{k_j} \to r_j$ was set for indexing without restriction of generality. This convention is retained in the following. It follows from equation (16) that the difference between the radius squares of neighboring interference rings is constant:

$$(17)\quad r_{j+1}^2 - r_j^2 = \frac{2\cdot f^2}{k_0} = \text{const.}.$$

From equations (16) and (17) follows:

$$(18)\quad \varepsilon = \frac{r_{j+1}^2}{r_{j+1}^2 - r_j^2} - j.$$

If the interference rings are split into two very closely spaced components a and b, whose wavelengths differ only slightly from each other, for the first interference ring counted from the center, for example, follows from equation (14):

$$(19)\quad \begin{aligned} \varepsilon_a &= k_{0,a} - k_{1,a} = \frac{2\cdot d\cdot n}{\lambda_a} - k_{1,a} \\ \varepsilon_b &= k_{0,b} - k_{1,b} = \frac{2\cdot d\cdot n}{\lambda_b} - k_{1,b} \end{aligned}$$

Since the two components belong to the same interference order, and provided that the interference rings do not overlap by more than one whole order, $k_{1,a} = k_{1,b}$ and thus:

$$(20)\quad \varepsilon_a - \varepsilon_b = k_{0,a} - k_{0,b} = 2\cdot d\cdot n\cdot\left(\frac{1}{\lambda_a} - \frac{1}{\lambda_b}\right).$$

Equation (20) does not explicitly depend on the interference order. If equation (18) is formulated for both components a and b and inserted into equation (20), the following results:

$$(21)\quad \left(\frac{1}{\lambda_a} - \frac{1}{\lambda_b}\right) = \frac{1}{2\cdot d\cdot n}\cdot\left(\frac{r_{j+1,a}^2}{r_{j+1,a}^2 - r_{j,a}^2} - \frac{r_{j+1,b}^2}{r_{j+1,b}^2 - r_{j,b}^2}\right).$$

From equation (17) it follows that the difference of the radius squares of the component a or b for neighboring interference orders j and j+1 with $j > 0$ due to $\lambda_a \approx \lambda_b$ and thus $k_{0,a} \approx k_{0,b}$ are approximately equal:

$$(22)\quad \Delta_a^{j+1,j} = r_{j+1,a}^2 - r_{j,a}^2 = r_{j+1,b}^2 - r_{j,b}^2 = \Delta_b^{j+1,j}.$$

Accordingly, the following applies for two components a and b of the same interference order j with $j > 0$:

$$(23)\quad \delta_{a,b}^{j} = r_{j,a}^2 - r_{j,b}^2 = r_{j+1,a}^2 - r_{j+1,b}^2 = \delta_{a,b}^{j+1}.$$

Substituting equations (22) and (23) into equation (21) results in:

$$(24)\quad \left(\frac{1}{\lambda_a} - \frac{1}{\lambda_b}\right) = \frac{1}{2\cdot d\cdot n}\cdot\frac{\delta_{a,b}^{j+1}}{\Delta_a^{j+1,j}} \text{ for all } j > 0$$

Since equation (22) applies to both components a and b of neighboring interference rings and equation (23) applies to all interference rings, mean values

$$(25)\quad \delta = \overline{\delta_{a,b}^{j}}$$

and

$$(26)\quad \Delta = \overline{\Delta_a^{j+1,j}}$$

can be calculated and inserted into equation (24):

$$(27)\quad \left(\frac{1}{\lambda_a} - \frac{1}{\lambda_b}\right) = \frac{1}{2\cdot d\cdot n}\cdot\frac{\delta}{\Delta}.$$

With

$$(28)\quad \Delta E_{a,b} = h\cdot c\cdot\left(\frac{1}{\lambda_a} - \frac{1}{\lambda_b}\right) = \mu_B\cdot B$$

follows from equation (27):

$$(29)\quad \frac{\delta}{\Delta} = 2\cdot\frac{d\cdot n}{h\cdot c}\cdot\mu_B\cdot B = a\cdot B \text{ with } a = 2\cdot\frac{d\cdot n}{h\cdot c}\cdot\mu_B.$$

The ratio δ / Δ can be measured as a function of the magnetic flux density B, plotted graphically, and the Bohr magneton μ_B can be determined from the slope a of a linear fit.

EQUIPMENT LIST

1 Cadmium Lamp with Accessories @230 V	1021366
or	
1 Cadmium Lamp with Accessories @115 V	1021747
1 Fabry-Pérot-Etalon 644 nm	1020903
1 DC Power Supply, linear regulated, 1 – 30V, 0 – 10A @230V	1025380
oder	
1 DC Power Supply 20 V, 5 A @115 V	1003311
1 U Core D	1022663
2 Coils D, 900 Turns	1012859
1 Electromagnet Accessory for Zeeman Effect	1021365
1 Microscope Camera Bresser MikroCam SP 3.1	1024060
1 Lens 12 mm for Bresser Microscope Camera	1024059
1 Stainless Steel Rod with ¼ inch Thread, 100 mm	1025431
1 Red Filter mounted on holder	1025376
2 Convex Lenses on Stem, *f* = 100 mm	1003023
1 Quarter-Wavelength Filter on Stem	1021353
1 Polarising Attachment	1021364
1 Polarisation Filter on Stem	1008668
1 Optical Precision Bench D, 1000 mm	1002628
1 Support for Optical Bench D	1012399
1 Optical Base D	1009733
3 Optical Riders D 90/36	1012401
2 Optical Riders D 60/36	1002639
1 Pair of Safety Experimenz Leads, 75 cm, blue, red	1017718
1 Pair of Safety Experimenz Leads, 75 cm, black	1002849

SETUP AND SAFETY INSTRUCTIONS

The performance of this experiment assumes that the assembly of the components as well as the experimental setup and adjustment have been carried out according to the instructions for the experiment UE5020850, considering all the safety instructions formulated therein.

The maximum current through the coils D with 900 turns is 5 A (7 minutes). It can be doubled for short periods (30 seconds). The coils have an internal reversible thermal fuse which trips at a winding temperature of 85°C. The reset time is 10-20 minutes, depending on the ambient temperature.

- Carry out the measurement quickly enough to prevent the thermal fuse from tripping due to high currents flowing for too long.
- Do not operate the coils without a transformer core.

EXPERIMENT PROCEDURE

Measurement

- Establish the transversal configuration by rotating the electromagnet as described in the instruction manual for the experiment UE5020850.

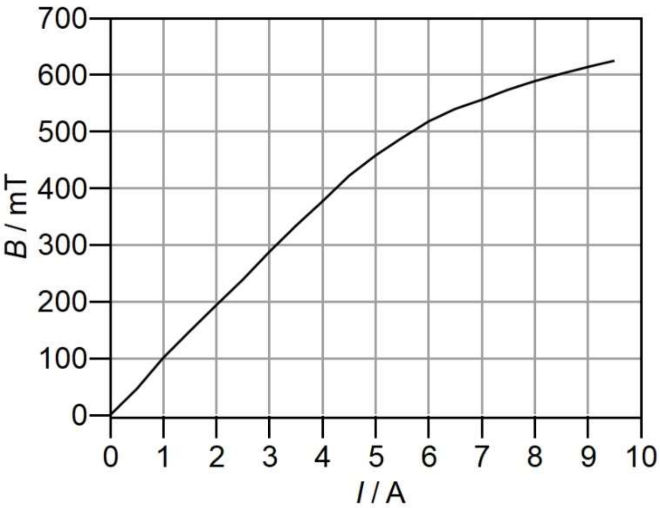

Fig. 4: Calibration curve of the electromagnet

- Focus the 12 mm lens so that the interference rings of the innermost order are in focus. Do not move the convex lenses (imaging and condenser lens) and do not refocus the 12 mm lens, otherwise the evaluation will give incorrect results.
- Switch on the DC power supply unit, increase the current through the coils first to 3 A, then in 0.5 A steps to 5 A and in 1 A steps to 9 A. At each step, take a screenshot ("snapshot") with the camera software and save it as a "JPEG".

Note:
When increasing the current, make sure that the interference rings do not overlap by more than one whole order.

Calibration of the electromagnet

The values for the magnetic flux densities *B*, which correspond to the set currents *I*, can be taken from the calibration curve in Fig. 4 or Tab. 1. Alternatively, the calibration curve can be measured as follows:

- Remove the Cd lamp on the housing from the base plate.
- Place a teslameter in the air gap between the two pole pieces (approx. 10 mm) so that the magnetic field sensor is centered.
- Switch on the DC power supply unit and increase the current *I* through the coils in 0.5 A steps. At each step, measure the values for the magnetic flux density *B*, note them and plot them against the set currents.
- Reduce the current to zero and switch off the DC power supply unit.
- Insert the Cd lamp back into the base plate.

Tab 1: Calibration of the electromagnet. Set currents *I* and measured magnetic flux densities *B*

I / A	*B* / mT	*I* / A	*B* / mT
0.0	0	5.0	458
0.5	46	5.5	489
1.0	101	6.0	518
1.5	148	6.5	540
2.0	194	7.0	556
2.5	239	7.5	574
3.0	288	8.0	589
3.5	334	8.5	602
4.0	377	9.0	614
4.5	422	9.5	625

MEASUREMENT EXAMPLE AND EVALUATION

The following steps are to be carried out for each saved screenshot:

- Open a screenshot in the camera software (click on “File” in the menu bar and select “Open image”).
- Click on “Options” in the menu bar, then on “Measurement”, select “Length Unit” in the window that opens, tick “Pixel” under “Current” and confirm the setting by clicking on “OK”.
- Click on the “Circle” button in the tool bar and select “3 Points”. Place a circle on the innermost interference ring. This is referred to as “C1” in the following.

The “Measurement” window opens automatically.

- If necessary, adjust the appearance under “Appearance” (e.g. line width/color, show/hide label type).
- Under “Geometry”, note the numerical value for the area in pixels (Tab. 2). Mark further interference rings in the same way (C2-C9, Fig. 5) and note the areas (Tab. 2). Click on the “Track” button (hand symbol) to complete the process.
- Click on “Layer” in the menu bar, select “Merge to image” and click on “OK”.
- Click on “File” in the menu bar, select “Save as” and save the image as a JPEG with a meaningful name.

Note:
The unit of the area is irrelevant for further evaluation, as only relative values and ratios are calculated, not absolute values. The absolute values of the areas (Tab. 2) can deviate significantly depending on the position of the optics.

- Calculate the area differences Δ of the corresponding components of neighboring interference orders (Eq. (22), Tab. 3; circles C4↔C1, C5↔C2, C6↔C3, C7↔C4, C8↔C5, C9↔C6).
- Calculate the area differences δ of neighboring components of the same interference orders (Eq. (23), Tab. 4; circles C2↔C1, C3↔C2, C5↔C4, C6↔C5, C8↔C7, C9↔C8).
- Calculate the mean values from all area differences in Tab. 3 and 4 (Eq. (25), (26)) and enter it in the tables.
- Calculate the ratio δ / Δ of the mean values for all set currents or magnetic flux densities, respectively (Tab. 5). Take the corresponding values for the magnetic flux density from the calibration curve of the electromagnet (Fig. 4, Tab. 1).
- Plot the ratio δ / Δ as a function of the magnetic flux density *B* and fit a straight line through the origin (Fig. 6).

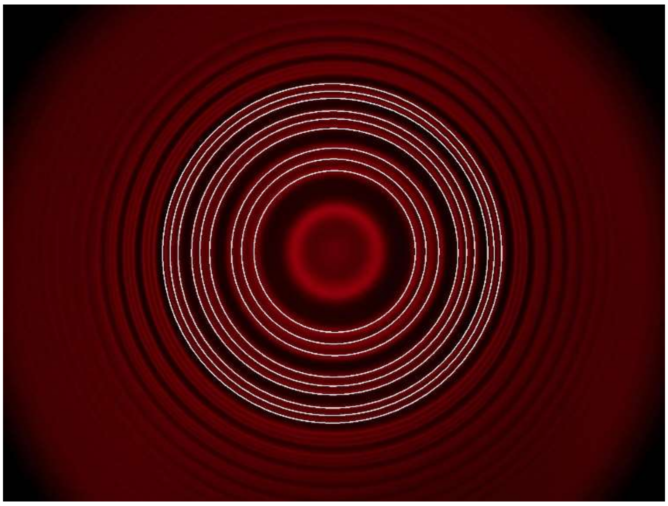

Fig. 5: Triplet splitting of the red cadmium line (I = 5.0 A ≙ B = 458 mT). Interference rings marked with circles to determine the enclosed areas

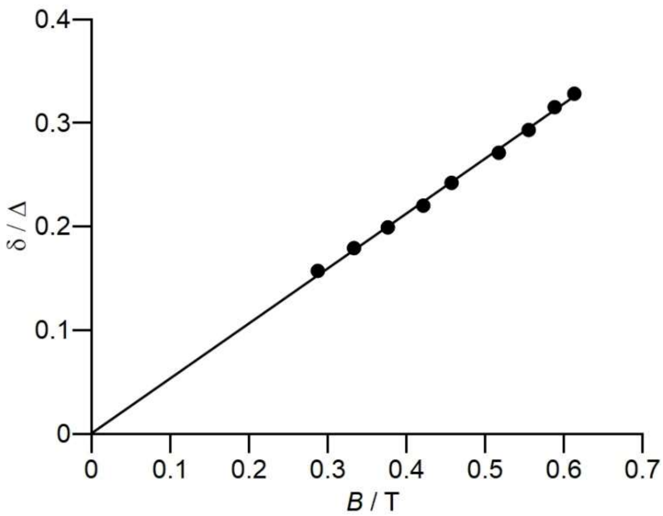

Fig. 6: Ratio δ / Δ of the area differences as a function of the magnetic flux density *B*. The slope of the fitted straight line through the origin is a = 0.53 / T

- Determine the Bohr magneton from the slope a = 0.53 / T of the fitted straight line using equation (29):

$$\mu_B = \frac{1}{2} \cdot \frac{h \cdot c}{d \cdot n} \cdot a$$

$$(30) \quad = \frac{1}{2} \cdot \frac{6.6 \cdot 10^{-34}\,\text{Js} \cdot 3.0 \cdot 10^{8}\,\text{m/s}}{4\,\text{mm} \cdot 1.4567} \cdot 0.53\,/\,\text{T}\ .$$

$$= 9.0 \cdot 10^{-24}\,\frac{\text{J}}{\text{T}}$$

The value corresponds to the literature value $9.3\ 10^{24}$ J/T except for approx. 3%.

Tab. 2: Areas *A* enclosed by the interference rings determined with the help of the camera software

I / A	Area *A* / Pixel								
	C1	C2	C3	C4	C5	C6	C7	C8	C9
3.0	167734	200055	229205	367830	398701	430412	559306	592777	620040
3.5	161486	200196	234474	365742	400854	434853	554225	592457	622683
4.0	157753	199493	238088	358148	398737	439637	552909	592559	624921
4.5	151447	200768	241074	354744	399174	442546	548700	591057	629975
5.0	146500	201657	248223	352695	398436	448720	546544	591877	633671
6.0	140903	199539	254920	345700	400889	451353	539028	591891	637638
7.0	134134	199027	257459	340850	401293	454900	535505	591126	643582
8.0	131146	199745	261665	335577	400627	460375	532289	591173	647816
9.0	130739	200385	265108	332857	398694	463757	531064	590470	651822

Tab. 3: Area differences Δ of the corresponding components of neighboring interference orders

I / A	Area difference Δ / Pixel						Mittelwert
	$\Delta_{C4,C1}$	$\Delta_{C5,C2}$	$\Delta_{C6,C3}$	$\Delta_{C7,C4}$	$\Delta_{C8,C5}$	$\Delta_{C9,C6}$	
3.0	200096	198646	201207	191476	194076	189628	195855
3.5	204256	200658	200379	188483	191603	187830	195535
4.0	200395	199244	201549	194761	193822	185284	195843
4.5	203297	198406	201472	193956	191883	187429	196074
5.0	206195	196779	200497	193849	193441	184951	195952
6.0	204797	201350	196433	193328	191002	186285	195533
7.0	206716	202266	197441	194655	189833	188682	196599
8.0	204431	200882	198710	196712	190546	187441	196454
9.0	202118	198309	198649	198207	191776	188065	196187

Tab. 4: Area differences δ of neighboring components of the same interference orders

I / A	Area difference δ / Pixel						Mean value
	$\delta_{C2,C1}$	$\delta_{C3,C2}$	$\delta_{C5,C4}$	$\delta_{C6,C5}$	$\delta_{C8,C7}$	$\delta_{C9,C8}$	
3.0	32321	29150	30871	31711	33471	27263	30798
3.5	38710	34278	35112	33999	38232	30226	35093
4.0	41740	38595	40589	40900	39650	32362	38973
4.5	49321	40306	44430	43372	42357	38918	43117
5.0	55157	46566	45741	50284	45333	41794	47479
6.0	58636	55381	55189	50464	52863	45747	53047
7.0	64893	58432	60443	53607	55621	52456	57575
8.0	68599	61920	65050	59748	58884	56643	61807
9.0	69646	64723	65837	65063	59406	61352	64338

Tab. 5: Ratio δ / Δ of the area differences for different currents *I* or magnetic flux densities *B*, respectively

I / A	*B* / T	δ / Δ
3.0	0.288	0.157
3.5	0.334	0.179
4.0	0.377	0.199
4.5	0.422	0.220
5.0	0.458	0.242
6.0	0.518	0.271
7.0	0.556	0.293
8.0	0.589	0.315
9.0	0.614	0.328